

Steady-state dynamics and effective temperatures of quantum criticality in an open system

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We study the thermal and non-thermal steady state scaling functions and the steady-state dynamics of a model of local quantum criticality. The model we consider, *i.e.* the pseudogap Kondo model, allows us to study the concept of effective temperatures near fully interacting as well as weak-coupling fixed points. In the vicinity of each fixed point we establish the existence of an effective temperature –different at each fixed point– such that the equilibrium fluctuation-dissipation theorem is recovered. Most notably, steady-state scaling functions in terms of the effective temperatures coincide with the equilibrium scaling functions. This result extends to higher correlation functions as is explicitly demonstrated for the Kondo singlet strength. The non-linear charge transport is also studied and analyzed in terms of the effective temperature.

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The interest in understanding the dynamics of strongly correlated systems beyond the linear response regime has in recent years grown tremendously.

The quantum dynamics in adiabatically isolated optical traps has been successfully modeled using powerful numerical schemes [1, 2]. In open systems mainly diagrammatic techniques on the Schwinger-Keldysh contour have been employed. For nanostructured systems several techniques exist to describe the ensuing out-of-equilibrium properties. These approaches, however, are either perturbative in nature [3], centered around high temperatures and short times [4–8], or approximate the continuous baths by discrete Wilson chains [9–11]. The situation might be simpler for non-linear dynamics that arises in the vicinity of a quantum critical point (QCP), where a vanishing energy scale leads to scaling and universality [12–19].

For the dynamics near classical continuous phase transitions a well-established theoretical framework exists, tying the dynamics to the statics and the conserved quantities [20]. In addition, the concept of effective temperature (T_{eff}) was established as an useful notion for the relaxational dynamics of classical critical systems [21–24], although it appears somewhat less useful for fully interacting critical points [23]. T_{eff} is commonly defined by extending the equilibrium fluctuation-dissipation theorem to the non-linear regime. The existence of effective temperatures in quantum systems was recently investigated [18, 25–28]. For a recent review see [25]. In comparison to classical criticality, at a QCP, dynamics already

enters at the equilibrium level. For a QCP that can be described by a Ginzburg-Landau-Wilson functional in elevated dimensions, it was found that the voltage-driven transition is in the universality class of the associated thermal classical model with voltage acting as T_{eff} [12]. Unconventional QCPs in contrast are not described solely in terms of an order parameter functional [29, 30].

In this letter we address the following general questions within a model system of unconventional quantum criticality: Is the existence of T_{eff} tied to dynamical (or ω/T -)scaling? Does T_{eff} have meaning for higher correlation functions? How unique is T_{eff} at a given fixed point once boundary conditions have been specified? Can critical scaling functions be expressed through T_{eff} and if so, how do these scaling functions relate to the equilibrium scaling functions? The model system is the pseudogap Kondo (pKM) model that describes a quantum spin antiferromagnetically coupled to a conduction electron bath possessing a pseudogap near its Fermi energy, characterized by a powerlaw exponent. Depending on the coupling strength, the quantum spin is either screened or remains free in the zero temperature (T) limit. The two phases are separated by a critical point displaying critical Kondo destruction, see Fig.1. The pKM has been invoked to describe non-magnetic impurities in the cuprate superconductors [31] and point-defects in graphene [32]. It underlies the pseudogap free moment phases occurring in certain disordered metals [33] and can also be realized in double quantum-dot systems [34]. The quantum critical properties of the pKM in equilibrium have been ad-

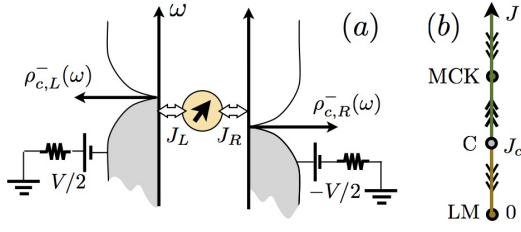


Figure 1. (a) Sketch of the model: a spin interacts with two fermionic leads which are characterized by their respective density of states $\rho_{c,L/R}(\omega)$ and chemical potential $\mu_{L/R}$. (b) Phase diagram of the multichannel pKM with gap exponent $r < r_{\max}$: A QCP (C) separates the multichannel Kondo fixed point (MCK) from the (weak-coupling) local moment fixed point (LM).

dressed in [35–45]. Our main findings are that the steady state dynamic spin susceptibility, the conductance, and the Kondo-singlet strength, a 4-point correlator, reproduce their equilibrium behavior in the scaling regimes of the fixed points of the model when expressed in terms of a fixed-point specific T_{eff} .

The model. We consider a pKM with a density of states that vanishes in a power-law fashion with exponent $0 \leq r \leq 1$ at their respective Fermi level, $\rho_{c,l}(\omega) \sim |\omega|^r \Theta(D - |\omega|)$, with half-bandwidth D . Here, $l = L, R$ labels the two leads, see Fig.1(a). In the multichannel version of the model the spin degree of freedom (\mathbf{S}) is generalized from $SU(2)$ to $SU(N)$ and the fermionic excitations (c) of the leads transform under the fundamental representation of $SU(N) \times SU(M)$ with N spin and M charge channels. At $T = 0$ and $r < r_{\max} < 1$, a critical point (C) separates a multichannel Kondo (MCK)-screened phase from a local moment (LM) phase at a critical value J_c of the exchange coupling $J > 0$, see Fig.1(b). The characterization of the phases and the leading power law exponents of observables of the pKM have been obtained by perturbative RG, large- N methods, and NRG [35–38, 40, 43]. Within the large- N approach, at $T = 0$, scaling arguments are able to predict the critical exponents of dynamical observables [39, 46]. Non-equilibrium steady-states (NESS) are obtained by applying a time-independent bias voltage $V = (\mu_L - \mu_R)/|e|$, where μ_l is the chemical potential of lead l , see Fig. 1(a). As T characterizes the fermionic reservoirs, it remains well-defined even for $V \neq 0$.

A similar setup has been considered in a perturbative RG-like study adapted to the NESS condition [47]. This model has also been invoked in a variational study of the dynamics following a local quench where it was found that quenches in the Kondo phase thermalize while this is not the case for quenches across the QCP into the LM regime [48]. The system is described by the Hamiltonian

$$H = \sum_{p,\alpha,l} \varepsilon_{pl} c_{p\alpha l}^\dagger c_{p\alpha l} + \frac{1}{N} \sum_{ll'} \sum_{\alpha} J_{ll'} \mathbf{S}_{\alpha;ll'}, \quad (1)$$

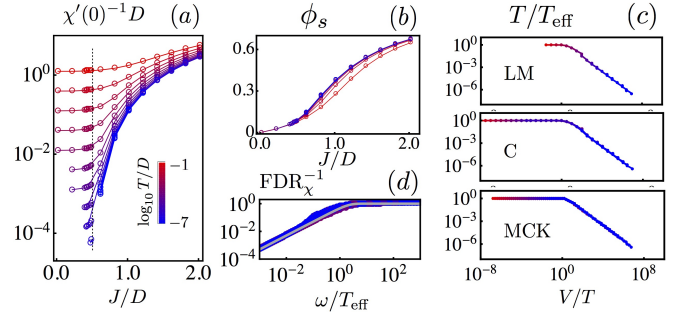


Figure 2. (a) $\chi'(0)^{-1}$ vs J for different T . (b) ϕ_s vs J for different T . The $T = 0$ curve is approached from below in the MCK and from above in the LM phases. (c) Scaling T/T_{eff} vs V/T at fixed points LM, C, and MCK: $T_{\text{eff}} \sim V$ for $V \gg T$. (d) $\text{FDR}_\chi^{-1}(\omega)$ vs ω/T_{eff} near fixed point C, shown for $V/D = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}$. The grey line is $\text{FDR}_\chi^{-1}(\omega) = \tanh(\beta\omega/2)$.

where $\sigma = 1, \dots, N$ and $\alpha = 1, \dots, M$ are, respectively, the $SU(N)$ -spin and $SU(M)$ -channel indices, l labels the leads and p is a momentum index. The co-tunneling term [49] in Eq. (1) contains the local operators $\mathbf{s}_{\alpha;ll'}^i = \frac{1}{n_c} \sum_{pp'\sigma\sigma'} c_{p\alpha\sigma l}^\dagger t_{\sigma\sigma'}^\dagger c_{p'\alpha\sigma' l'}$ with t the fundamental representation of $SU(N)$ and n_c is the number of fermionic single-particle states. In a totally anti-symmetric representation, one can decompose the spin operator as $S_{\sigma\sigma'} = f_\sigma^\dagger f_{\sigma'} - q \delta_{\sigma\sigma'}$, where q is subject to the constraint $\hat{Q} = \sum_{\sigma} f_\sigma^\dagger f_\sigma = qN$ and the $f_\sigma^\dagger, f_{\sigma'}$ obey fermionic commutation relations.

We employ a dynamical large- N limit [39, 50], suitably generalized to the Keldysh contour [16, 18] while keeping $q = \frac{Q}{N}$ and $\kappa = M/N$ constant. This results in

$$\Sigma_B^{>,<}(t) = iG_f^{>,<}(t) G_c^{<,>}(-t) \quad (2)$$

$$\Sigma_f^{>,<}(t) = -i\kappa G_B^{>,<}(t) G_c^{>,<}(t) \quad (3)$$

$$-iG_f^{<}(0) = q, \quad (4)$$

where G_f is the pseudofermion propagator and G_B is the propagator of a bosonic Hubbard-Stratonovich decoupling field. Σ_f (Σ_B) is the proper selfenergy of G_f (G_B) and is related to it via the Dyson equation [51]. We assume that the exchange interaction originates from an Anderson-type model via a Schrieffer-Wolff transformation, so that a single coupling constant $J = J_L + J_R$ emerges [51].

For details on the numerics see [51]. In equilibrium, our approach yields dynamical scaling functions that coincide with those obtained from quantum Monte-Carlo [44].

Observables. A possible order parameter for the transition from the overscreened Kondo to local-moment phase is given by $\lim_{T \rightarrow 0} T\chi(\omega = 0, T)$, where $\chi(\omega, T)$ is the Fourier transform of the local (impurity) spin-spin correlation function $\chi(t - t')$, see Fig. 2(a). We work on the Keldysh contour where the lesser and greater components are defined in the usual way as $\chi^>(t - t') =$

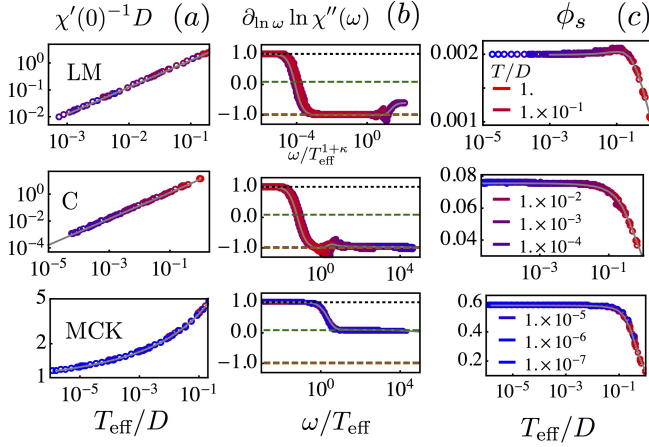


Figure 3. Scaling of observables with T_{eff} at different fixed points for the values of V as in Fig. 2(d): (a) $\chi'(0)^{-1}$ vs T_{eff} ; (b) $\partial_{\ln \omega} \ln \chi''(\omega)$ vs ω/T_{eff} ; (c) ϕ_s vs T_{eff} . For each fixed point, the equilibrium scaling form (grey curves) is compared with the same quantity under non-equilibrium conditions and T substituted by T_{eff} .

$-i \frac{1}{N} \sum_a \langle S^a(t) S^a(t') \rangle$ with $t \in \gamma_{\leftarrow}$ and $t' \in \gamma_{\rightarrow}$ and $\chi^<(t-t') = -i \frac{1}{N} \sum_a \langle S^a(t') S^a(t) \rangle$, with $t \in \gamma_{\rightarrow}$ and $t' \in \gamma_{\leftarrow}$ so that $\chi^R(t) = \Theta(t) [\chi^>(t) - \chi^<(t)]$ and $\chi^A = \chi^R + \chi^< - \chi^>$. Here, $\gamma_{\rightarrow(\leftarrow)}$ is the forward (backward) branch of the Keldysh contour, respectively.

We also consider the “singlet-strength” ϕ_s , defined through the Kondo term contribution to the total energy of the system as $\frac{1}{N} \sum_{ll'} \sum_c J_{ll'} \langle \mathbf{S} \cdot \mathbf{s}_{c, ll'} \rangle = -J\kappa \left(\frac{N^2-1}{N} \right) \phi_s$ [52]. ϕ_s is a dimensionless quantity, which possesses a well-defined large- N limit and quantifies the degree of singlet formation. In terms of the fermionic fields, it can be written as the local-in-time limit of a 4-point correlator [51]. Its equilibrium properties will be discussed below. The steady state charge current passing through each channel is $\mathcal{J}_P = -\partial_t \langle \hat{N}_L(t) \rangle / M$, where $\hat{N}_L = \sum_{p\alpha\sigma} c_{p\alpha\sigma L}^\dagger c_{p\alpha\sigma L}$ is the number of particles in the left lead. The out-of-equilibrium conditions considered here respect particle-hole symmetry which implies a vanishing energy current.

Throughout the paper we set $\kappa = 0.3$, $r = 0.2$, and $q = 1/2$. This results in $r_{\text{max}} = 0.412(4)$. Our choice of values for κ and r ensures a finite static spin susceptibility $\chi'(\omega = 0)$ within the MCK phase as $T \rightarrow 0$. We denote the real (imaginary) part of $\chi^R(\omega)$ by χ' (χ''). *Thermal steady-state.* The equilibrium ($V = 0$) behavior of $\chi'(\omega = 0, T)$ in the relaxational regime ($\omega \ll T$) near the MCK, C, and LM fixed points is shown in Fig. 2-(a). For $J < J_c \simeq 0.44D$, i.e. in the LM phase, one observes Curie-like behavior at lowest temperatures $\chi'(\omega = 0, T) \propto T^{-1}$. In the MCK phase ($J > J_c$ and with our choices of κ and r), the $T = 0$ susceptibility remains finite. The grey lines in Figs. 3-(b) show the scaling plots of the logarithmic derivative of $\chi''(\omega)$ for different

values of the temperature, i.e. $\partial_{\ln \omega} \ln \chi''(\omega)$ for the different fixed points. Note that $\partial_{\ln \omega} \ln \chi''(\omega) \simeq \alpha_\chi$ within the scaling region where $\chi''(\omega) \propto |\omega|^{\alpha_\chi}$. The values of α_χ in the quantum coherent regime ($\omega/T \gg 1$) agree with those obtained analytically from a $T = 0$ scaling ansatz [46] for the MCK ($\alpha_\chi \simeq 0.087$) and C ($\alpha_\chi = -0.97$) fixed points. These results are compatible with a dynamical scaling form $\chi''(\omega, T) = T^{\alpha_\chi} \Phi(\omega/T)$, in terms of an universal scaling function $\Phi(x)$ possessing asymptotic values $\Phi(x) \propto x$ for $x \ll 1$ and $\Phi(x) \propto x^{\alpha_\chi}$ for $x \gg 1$. Thus, the scaling properties are in line with dynamical ω/T -scaling for the C and MCK fixed points. For the LM fixed point we find $\alpha_\chi = -1$ and a scaling form $\chi''(\omega) = T^{\alpha_\chi} \hat{\Phi}(\omega/T^{1+\kappa})$, indicative of a weak-coupling fixed point and absence of hyperscaling. These results will be further addressed elsewhere [46].

The singlet-strength ϕ_s vs. J at different T and at $V = 0$ is shown in Fig. 2-(b). The numerical data at $T \neq 0$ suggest that $\phi_s(J, T = 0)$ is a continuous function of J . At the C fixed point we find that $\phi_s(J, T)$ as a function of J crosses for different values of T (for sufficiently low T). *Non-thermal steady-states.* We consider a non-equilibrium setup where the two leads, initially decoupled from the impurity (for $t < t_0$), are held at chemical potentials $\mu_L = -\mu_R = |e|V/2$ ($|e| = 1$ in the following). At $t = t_0$ the coupling between the leads and the impurity is turned on. A steady-state is reached by sending $t_0 \rightarrow -\infty$ so that any transient behavior will already have faded away at (finite) time t . The NESS fluctuation-dissipation ratio (FDR) for a dynamical observable $A(t, t') = A(t - t')$ is defined through $\text{FDR}_A(\omega) = [A^>(\omega) + A^<(\omega)]/[A^>(\omega) - A^<(\omega)]$, where $A^{>/<}$ are the Fourier transforms of the greater/lesser components of A . At equilibrium, the fluctuation-dissipation theorem implies $\text{FDR}_A(\omega) = \tanh(\beta\omega/2)^\zeta$ uniquely (with $\zeta = \pm 1$ for fermionic (+) and bosonic (-) operators). For a generic out-of-equilibrium system, the functional form of the FDR differs from the equilibrium one. A frequency-dependent “effective temperature”, $1/\beta_{\text{eff}}^A(\omega)$, for the observable A can be defined by requiring that $\tanh[\beta_{\text{eff}}^A(\omega)\omega/2]^\zeta = \text{FDR}_A(\omega)$ [27, 53]. Following Refs. [18, 21, 26] we define T_{eff} via FDR_χ through its asymptotic low-frequency behavior $T_{\text{eff}}^{-1} = \lim_{\omega \rightarrow 0} \beta_{\text{eff}}^\chi(\omega)$. In equilibrium $T_{\text{eff}} = T$. On the other hand, a linear-in- V decoherence rate in the non-equilibrium relaxational regime near an interacting QCP is signaled by ω/V -scaling [16]. In this case and at $T = 0$ one expects $T_{\text{eff}} = cV$, where c characterizes the underlying fixed point. We thus analyze T/T_{eff} vs V/T . Fig. 2-(c) shows the resulting T/T_{eff} as a function of V/T for the different fixed points computed for different values of V and T . In the non-linear regime, the scaling collapse for T/T_{eff} implies $T_{\text{eff}} = cV$, where $1/c$ is the amplitude of the scaling curve in the non-linear regime. A comparison of FDR_χ^{-1} with the equilibrium result for fixed point (C)

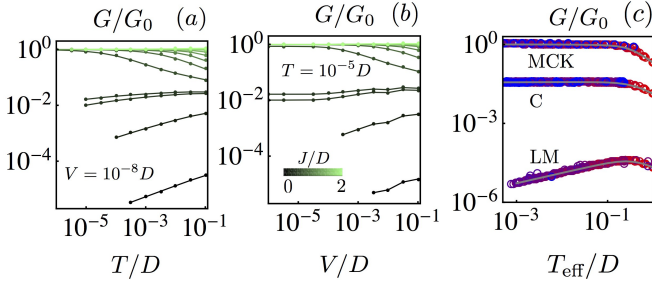


Figure 4. Conductance G normalised to the MCK fixed point conductance $2\pi G_0 = 0.415$. (a) $G(T)$ vs T computed for the lowest non-zero value of V at different values of J (see color coding). (b) G vs V for fixed T . (c) $G = \mathcal{J}_P/V$ vs T_{eff} at different fixed points. The equilibrium form is given by the grey curves.

is shown in Fig.2-(d). Even for the LM fixed point, where hyperscaling is violated, $T_{\text{eff}} \sim V$ holds for $V \gg T$, see Fig. 2-(c), top panel. It is however important to realize that the properties we see in terms of T_{eff} are a property of the flow towards the LM fixed point. Far from equilibrium and outside any scaling regime, χ is a function of ω , T , and V but near a fixed point $\chi(\omega, T, V)$ develops a scaling form in terms of a combination of ω , T , and V . This then raises the question how T_{eff} enters the scaling function and leads us to a remarkable result, see Fig. 3-(b)-(c): The non-thermal steady-state scaling function of $\chi = \chi(\omega, T, V)$ when scaled in terms of T_{eff} recovers the equilibrium scaling function of that particular fixed point with T_{eff} replacing T . This not only turns out to be true for χ at each of the fixed points of the model but also holds for ϕ_s , a higher-order correlation function. We first consider the static susceptibility. Fig. 3-(a) shows the equilibrium scaling forms of $\chi'(0)^{-1}$ as a function of T_{eff} for different values of T and V for the LM, C and MCK fixed points. The color coding reflects the values of T of the system. The equilibrium form (grey lines) is recovered even for $T_{\text{eff}}/T \gg 1$.

A similar result can be obtained at finite ω : Fig. 3-(b) shows the log-derivative $\partial_{\ln \omega} \ln \chi''(\omega)$ as a function of ω/T_{eff} for different values of T and V for the LM, C and MCK fixed points. These should be compared with the equilibrium results, the underlying grey lines: The equilibrium scaling form is recovered by replacing T by T_{eff} , both for $\omega \ll T_{\text{eff}}$ and $\omega \gg T_{\text{eff}}$ [54]. Note that T_{eff} is defined from the FDR of χ in the limit $\omega/T \rightarrow 0$. Therefore, the fact that the equilibrium scaling forms of $\chi'(0)$ and $\chi''(\omega)$ are reproduced for $T_{\text{eff}}/T \gg 1$ and $\omega/T_{\text{eff}} \gg 1$, respectively, is remarkable. Fig. 3-(c) depicts ϕ_s as a function of T_{eff} for different values of T and V . Again, the equilibrium scaling behavior (gray curves) is reproduced.

Unlike χ and ϕ_s , the conductance G depends on both pseudoparticle propagators G_f and G_B . One thus may wonder if T_{eff} can have any meaning for G . In Figs. 4-

(a,b) we show the conductance per channel $G = \mathcal{J}_P/V$ vs T and V respectively. In the linear response regime $V, T \ll T_K(J)$ of the MCK phase, the current is proportional to the applied voltage $\mathcal{J}_P = G_0 V$. Outside of the scaling regime, i.e. for $V, T \gg T_K(J)$, G drops rapidly as V or T increase. The linear and non-linear current-voltage characteristics display power-law behavior as $T, V \rightarrow 0$ [16, 18]. Near C, i.e. for $J = J_c$, the relation between \mathcal{J}_P and V is still linear, ($\mathcal{J}_P = G_c V$), however the critical conductance G_c is much smaller than G_0 . Fig. 4-(c) shows G vs T_{eff} for different values of T and V for the LM, C and MCK fixed points. The grey curves are obtained by varying T at fixed V for the lowest value of V considered in our study, i.e. $V_{\text{min}} = 10^{-8} D$. The temperature dependence of the linear response conductance is reproduced at all fixed points when the non-linear conductance is taken as a function of T_{eff} . This is true even for values of V several orders of magnitude larger than V_{min} .

In conclusion, we have addressed the steady-state dynamics near unconventional quantum criticality. We found that in the scaling regime of all the fixed points considered, all observables studied (χ, ϕ_s, G) scale in terms of the same but fixed point specific effective temperature T_{eff} . The local spin-spin correlation function χ and the singlet-strength ϕ_s assume their equilibrium scaling functions even far from equilibrium when scaled in terms of T_{eff} , i.e. T_{eff} replacing T . A similar result relates the linear and non-linear conductance. We note that in the (non-interacting) pseudogap resonant level model such behavior is absent [46]. It has been shown that the non-equilibrium current noise near quantum criticality in models possessing gravity duals appears thermal [55, 56]. Our results imply that similar results hold for a larger class of quantum critical systems and quantities. The results reported here may thus help in identifying universality classes of unconventional quantum criticality. To which extend our results rely on locality needs to be further investigated.

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Steady-state dynamics and effective temperatures of quantum criticality in an open system: Supplementary Material

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GENERATING FUNCTIONAL ON THE KELDYSH CONTOUR

The generating functional on the Keldysh contour can be written as

$$\begin{aligned} Z[\xi] = & \int Dc \int Df \int D\lambda e^{i(c^\dagger g_c^{-1} c + f^\dagger g_f^{-1} f)} e^{iqN \int_\gamma dz \lambda_z} \\ & \times e^{+i\frac{1}{N} \int_\gamma dz \sum_\alpha J_{ll'z} (\sum_\sigma f_{\sigma,z}^\dagger c_{0,\alpha\sigma l'z}) (\sum_{\sigma'} c_{0\alpha\sigma' lz}^\dagger f_{\sigma'z})} \\ & \times e^{c^\dagger \xi_c + \xi_c^\dagger c + f^\dagger \xi_f + \xi_f^\dagger f} \end{aligned} \quad (\text{S.1})$$

where ξ_c and ξ_f act as sources to the fermionic c and f fields and λ is a scalar Lagrange multiplier enforcing the constraint $\sum_\sigma f_\sigma^\dagger f_\sigma = qN$. $\int_\gamma dz$ is the integral over the Keldysh contour $\gamma = \gamma_{\rightarrow} + \gamma_{\leftarrow}$ with its forward (γ_{\rightarrow}) and backward (γ_{\leftarrow}) branches.

Here, the inverse bare propagators are

$$g_f^{-1} = (i\partial_z - \lambda_z), \quad (\text{S.2})$$

$$g_c^{-1} = (i\partial_z - \varepsilon_{pl}). \quad (\text{S.3})$$

In analogy to the equilibrium procedure –albeit performed on the Matsubara contour– one can introduce a Hubbard-Stratonovich decoupling field $B_{\alpha lz}$ conjugated to $\sum_{\sigma'} c_{0\alpha\sigma' lz}^\dagger f_{\sigma'z}$, to decouple the quartic fermionic term in Eq.(S.1). Thus,

$$\begin{aligned} Z[\xi] = & \int Df \int D\lambda \int DB e^{i(f^\dagger g_f^{-1} f)} e^{i \int dz \lambda_z Q} e^{i(B_\alpha^\dagger g_B^{-1} B_\alpha)} \\ & \times e^{-i\frac{1}{N} \int dz \int dz' \sum_{\sigma\alpha l} B_{\alpha lz} B_{\alpha l'z}^\dagger \tilde{g}_{c,l}(z, z') f_{\sigma,z}^\dagger f_{\sigma,z'}} \\ & \times e^{\left[-\frac{1}{\sqrt{N}} \int_\gamma dz' \sum_{\alpha'\sigma'} (\xi_c^\dagger g_c)_{0\alpha'\sigma' l'z} B_{\alpha l'z'}^\dagger f_{\sigma'z'} - \frac{1}{\sqrt{N}} \int_\gamma dz \sum_{\sigma\alpha} f_{\sigma,z}^\dagger B_{\alpha lz} (g_c \cdot \xi_c)_{0\alpha\sigma lz}\right]} \\ & \times e^{i\xi_c^\dagger g_c \xi_c} e^{f^\dagger \xi_f + \xi_f^\dagger f} \end{aligned} \quad (\text{S.4})$$

with

$$g_{B;ll'}^{-1} = -\left[\tilde{J}^{-1}\right]_{ll'}, \quad (\text{S.5})$$

where $\left[\tilde{J}\right]_{ll'} = J_{ll'}$, is the bare inverse propagator of the B field.

Finally, with the help of the complex-valued dynamic Hubbard-Stratonovich fields $W_{zz';l}$ one obtains

$$\begin{aligned} Z[\xi] = & \int D\lambda \int DW e^{N \text{tr} \ln[-iG_f^{-1}] - M \text{tr} \ln[-i(G_B^{-1} + V_{\xi_c}^\dagger G_f V_{\xi_c})]} \\ & e^{iN \text{tr}[W^\dagger * [\tilde{g}_c]^{-1} * W] + i \int dz \lambda_z Q} \\ & e^{-i\xi_f^\dagger G_f V_{\xi_c} [G_B^{-1} + V_{\xi_c}^\dagger G_f V_{\xi_c}]^{-1} V_{\xi_c}^\dagger G_f \xi_f + i\xi_c^\dagger g_c \xi_c + i\xi_f^\dagger G_f \xi_f} \end{aligned} \quad (\text{S.6})$$

with

$$G_f^{-1}(z, z') = g_f^{-1}(z, z') - W_{zz';l} \quad (\text{S.7})$$

$$G_B^{-1}(z, z') = g_b^{-1}(z, z') - \bar{W}_{z'z;l} \quad (\text{S.8})$$

$$\text{tr} \left[W^\dagger * [\tilde{g}_c]^{-1} * W \right] = \sum_l \int dz \int dz' \frac{\bar{W}_{zz';l} W_{zz';l}}{\tilde{g}_{c,l}(z, z')} \quad (\text{S.9})$$

and $V_{\xi_c}^\dagger$ and V_{ξ_c} are source-dependent terms. Eq.(S.6) is used to derive all correlators by taking derivatives with respect to the source fields.

DYNAMICAL LARGE-N SELF-CONSISTENCY EQUATIONS ON THE KELDYSH CONTOUR

In this section we set the sources to zero and compute the saddle-point equations with respect to the bosonic fields W and λ . The generating functional in the absence of sources is

$$Z[\xi = 0] = \int D\lambda \int DW e^{iN S[W, \lambda]} \quad (\text{S.10})$$

with

$$S[W, \lambda] = q \int dz \lambda_z + \text{tr} [W^\dagger * [\tilde{g}_c]^{-1} * W] - i \frac{1}{N} \text{tr} \ln [-iG_f^{-1}] + i\kappa \frac{1}{M} \text{tr} \ln [-iG_B^{-1}]. \quad (\text{S.11})$$

The saddle point equations are obtained by putting the linear variation of $S[W, \lambda]$ with respect to W and λ to zero:

$$\frac{\delta}{\delta W_{zz',l}} S[W, \lambda] = \bar{W}_{zz',l} [\tilde{g}_c(z, z')]^{-1} + i \frac{1}{N} \sum_\sigma G_f(z', z) = 0, \quad (\text{S.12})$$

$$\frac{\delta}{\delta \bar{W}_{zz',l}} S[W, \lambda] = [\tilde{g}_{c,l}(z, z')]^{-1} W_{zz',l} - i\kappa \frac{1}{M} \sum_\alpha G_{B;ll}(z, z') = 0, \quad (\text{S.13})$$

$$\frac{\delta}{\delta \lambda_z} S[W, \lambda] = q + i \frac{1}{N} \sum_\sigma G_f(z^-, z) = 0. \quad (\text{S.14})$$

These equations become *exact* in the large-N limit. These equations are equivalent to

$$\begin{aligned} \hat{G}_f^{-1} &= \hat{g}_f^{-1} - \Sigma_f \\ \hat{G}_B^{-1} &= \hat{g}_B^{-1} - \Sigma_B \\ q &= -i\hat{G}_f(z^-, z) \end{aligned}$$

with

$$\Sigma_B(z, z') = \begin{pmatrix} \bar{W}_{z'z,L} & 0 \\ 0 & \bar{W}_{z'z,R} \end{pmatrix} = -i \begin{pmatrix} \tilde{g}_{c,L}(z', z) & 0 \\ 0 & \tilde{g}_{c,R}(z', z) \end{pmatrix} \hat{G}_f(z, z') \quad (\text{S.15})$$

$$\Sigma_f(z, z') = \delta_{\sigma\sigma'} \sum_l W_{zz',l} = \delta_{\sigma\sigma'} i\kappa \sum_l \tilde{g}_{c,l}(z, z') \hat{G}_{B;ll}(z, z'). \quad (\text{S.16})$$

Note that λ_z evaluated at the saddle-point is time independent, *i.e.* $\lambda_t = \lambda$.

SINGULAR EXCHANGE COUPLING MATRIX J

So far, the treatment has been general and no particular form of the Kondo exchange coupling matrix has been assumed. For the physically most relevant case where the Kondo Hamiltonian is derived from an Anderson-type model through a Schrieffer-Wolff transformation, the exchange matrix $J_{ll'}$ ($l, l' = L, R$) takes the form

$$J = \begin{pmatrix} J_L & \sqrt{J_L J_R} \\ \sqrt{J_L J_R} & J_R \end{pmatrix}.$$

Thus, the exchange coupling matrix is singular, $\det(J) = 0$. In this case, where one of the eigenvalues of J vanishes, we can write

$$\tilde{J} = |u_+\rangle (J_L + J_R) \langle u_+|$$

with

$$\begin{aligned} |u_-\rangle &= -\sqrt{\frac{J_R}{J_L + J_R}} |L\rangle + \sqrt{\frac{J_L}{J_L + J_R}} |R\rangle \\ |u_+\rangle &= \sqrt{\frac{J_L}{J_L + J_R}} |L\rangle + \sqrt{\frac{J_R}{J_L + J_R}} |R\rangle. \end{aligned}$$

As the exchange matrix is singular, the component u_- of the B field has to vanish and thus

$$\hat{G}_B = |u_+\rangle \hat{G}_{B+} \langle u_+|.$$

In this case the self-consistent equations simplify to

$$\Sigma_{B+}(z, z') = -i\tilde{g}_{c,+}(z', z)\hat{G}_f(z, z'), \quad (\text{S.17})$$

$$\Sigma_f(z, z') = i\kappa\tilde{g}_{c,+}(z, z')\hat{G}_{B+}(z, z'), \quad (\text{S.18})$$

with

$$\begin{aligned} \hat{G}_{B+}^{-1} &= \hat{g}_{B+}^{-1} - \Sigma_B, \\ \hat{g}_{B+}^{-1} &= \frac{-1}{J_L + J_R}, \\ \tilde{g}_{c,+} &= \frac{J_L\tilde{g}_{c,L} + J_R\tilde{g}_{c,R}}{J_L + J_R}. \end{aligned}$$

Using Langreth's rules, we obtain

$$\Sigma_B^{>,<}(t, t') = -i\tilde{g}_c^{<,>}(t', t)\hat{G}_f^{>,<}(t, t'), \quad (\text{S.19})$$

$$\Sigma_f^{>,<}(t, t') = i\kappa\tilde{g}_c^{>,<}(t, t')\hat{G}_B^{>,<}(t, t'), \quad (\text{S.20})$$

$$q = -i\hat{G}_f^{<}(0). \quad (\text{S.21})$$

NON-EQUILIBIRUM STEADY-STATE (NESS) DESCRIPTION

The steady-state condition implies that the system is time translationally invariant so that $G^{R,A,K}(t, t') = G^{R,A,K}(t - t')$. Therefore, it is advantageous to solve the self-consistent equations in the frequency domain. The conventions of the Fourier transform used by us are

$$\begin{aligned} A(t) &= \int \frac{d\omega}{2\pi} A(\omega) e^{-i\omega t}, \\ A(\omega) &= \int dt A(t) e^{i\omega t}. \end{aligned}$$

Eq.(S.19-S.21) take the form

$$\Sigma_B^{>,<}(\omega) = -i \int \frac{d\nu}{2\pi} \tilde{g}_c^{<,>}(\nu - \omega) G_f^{>,<}(\nu), \quad (\text{S.22})$$

$$\Sigma_f^{>,<}(\omega) = i\kappa \int \frac{d\nu}{2\pi} \tilde{g}_c^{>,<}(\omega - \nu) G_B^{>,<}(\nu), \quad (\text{S.23})$$

$$q = -i \int \frac{d\omega}{2\pi} G_f^{<}(\omega). \quad (\text{S.24})$$

The reservoirs are in equilibrium and are thus characterized by their respective chemical potentials μ_L and μ_R and their respective temperatures $T_L = T_R = T$. We introduce the following reservoir quantities

$$\rho_{c,l}^{\pm}(\omega) = -\frac{1}{2\pi i} \left[\tilde{g}_{c,l}^{>}(\omega) \pm \tilde{g}_{c,l}^{<}(\omega) \right], \quad (\text{S.25})$$

$$\rho_{c,l}^H(\omega) = -\frac{1}{\pi} P \int d\nu \frac{\rho_{c,l}^-(\nu)}{\omega - \nu}, \quad (\text{S.26})$$

where $\rho_{c,l}^-(\omega) = \frac{1}{L} \sum_p \delta(\omega - \varepsilon_{pl})$ is the normalized ($\int d\omega \rho_{c,l}^-(\omega) = 1$) local density of states of reservoir l , $\rho_{c,l}^H(\omega)$ is its Hilbert transformed and $\rho_{c,l}^+(\omega)$ is proportional to the Keldysh component of the Green's function. Since the reservoirs are taken to be in equilibrium, the fluctuation dissipation theorem can be applied and it is found that

$$\rho_{c,l}^+(\omega) = f_l(\omega) \rho_{c,l}^-(\omega), \quad (\text{S.27})$$

with

$$f_l(\omega) = [1 - 2n_{f_l}(\omega - \mu_l)] = \tanh \left[\frac{\beta_l}{2} (\omega - \mu_l) \right]. \quad (\text{S.28})$$

Here, $n_{f_l}(\omega - \mu_l)$ is the Fermi-function, and β_l and μ_l are the inverse temperature and the chemical potential of reservoir l . The lead's Green's functions can thus be written in the form

$$\tilde{g}_c^{R,A}(\omega) = -\pi [\rho_c^H(\omega) \pm i\rho_c^-(\omega)], \quad (\text{S.29})$$

$$\tilde{g}_c^K(\omega) = -2\pi i\rho_c^+(\omega), \quad (\text{S.30})$$

with

$$\rho_c^-(\omega) = \frac{J_L \rho_{c,L}^-(\omega) + J_R \rho_{c,R}^-(\omega)}{J_L + J_R}, \quad (\text{S.31})$$

$$\rho_c^+(\omega) = \frac{J_L f_L(\omega) \rho_{c,L}^-(\omega) + J_R f_R(\omega) \rho_{c,R}^-(\omega)}{J_L + J_R}. \quad (\text{S.32})$$

Self-consistent equations for the steady-state

With the definitions of the previous sections, Dyson's equation translates to

$$\begin{aligned} \rho_f^\pm(\omega) &= \sigma_f^\pm(\omega) \left\{ \left[\omega - \tilde{\lambda} + \pi \sigma_f^H(\omega) \right]^2 + \left[\pi \sigma_f^-(\omega) \right]^2 \right\}^{-1}, \\ \rho_B^\pm(\omega) &= \sigma_B^\pm(\omega) \left\{ \left[-(J_L + J_R) + \pi \sigma_B^H(\omega) \right]^2 + \left[\pi \sigma_B^-(\omega) \right]^2 \right\}^{-1}, \end{aligned}$$

with $\tilde{\lambda} = \lambda_t - \frac{\kappa}{2}(J_L + J_R) \int d\omega \rho_c^+(\omega)$ being a renormalized chemical potential, and Eq.(S.22-S.24) translate to

$$\sigma_B^\pm(\omega) = \mp \frac{1}{2} \int d\nu \left[\rho_c^\pm(\nu - \omega) \rho_f^+(\nu) - \rho_c^\mp(\nu - \omega) \rho_f^-(\nu) \right], \quad (\text{S.33})$$

$$\sigma_f^\pm(\omega) = \kappa \frac{1}{2} \int d\nu \left[\rho_c^\pm(\omega - \nu) \rho_B^+(\nu) + \rho_c^\mp(\omega - \nu) \rho_B^-(\nu) \right], \quad (\text{S.34})$$

$$q = \frac{1}{2} \left[1 - \int d\omega \rho_f^+(\omega) \right]. \quad (\text{S.35})$$

In the particle-hole symmetric case ($q = 1/2$) and for a particle-hole symmetric DOS of the leads ($\rho_c^-(\omega) = \rho_c^-(-\omega)$) the quantities $\rho_{f,B}^\pm$ and $\sigma_{f,B}^\pm$ are real.

Details of the numerical treatment

The explicit form of the pseudogap density of states of the leads is taken to be

$$\rho_{c,l}^-(\omega) = \frac{1}{\sqrt{2}\Lambda \Gamma\left(\frac{r+1}{2}\right)} \left| \frac{\omega}{\sqrt{2}\Lambda} \right|^r e^{-\frac{\omega^2}{2\Lambda^2}},$$

with $l = R, L$ and $\Lambda = 1$ specifies the soft high-energy cutoff. The self-consistent equations were solved iteratively on a logarithmically discretized grid with 350 points ranging from -10Λ to 10Λ . The criterium for convergence of the selfconsistency loop was that the relative difference of two consecutive iterations was less than 10^{-6} . The results were benchmarked by the conditions that the fluctuation dissipation ratios of the Green's functions have to accurately reproduce the equilibrium fluctuation dissipation relations demanded by the fluctuation-dissipation theorem. For all the fixed points we studied a range of temperatures $T/D = 10^{-1}, 10^{-1.5}, 10^{-2}, \dots, 10^{-8}$ and a range of voltages $T/D = 10^{-2}, 10^{-1.5}, 10^{-2}, \dots, 10^{-8}$. However convergence of the numerical solution of the self-consistent equations was not always achieved for all combinations of parameters.

OBSERVABLES

Cross 4-point function

In order to compute the currents and the Kondo singlet strength we will need to evaluate the connected 4-point function $\langle T_\gamma c_{p_2\alpha_2\sigma_2l_2}^\dagger c_{p_1\alpha_1\sigma_1l_1} f_{s_2}^\dagger f_{s_1} \rangle_C$. Here, C denotes the connected part of a correlation function and T_γ is the time-ordering operator on the Keldysh contour. Using the procedure outlined above, one obtains

$$\langle T_\gamma f_{s_1}(t_1) f_{s_2}^\dagger(t_2) c_{p_1\alpha_1\sigma_1l_1}(t_3) c_{p_2\alpha_2\sigma_2l_2}^\dagger(t_4) \rangle_C = i \frac{1}{N} \frac{1}{n_c} \delta_{s_1\sigma_2} \delta_{s_2\sigma_1} \delta_{\alpha_1\alpha_2} F_{p_1l_1;p_2l_2}(t_1, t_2, t_3, t_4)$$

and

$$F_{p_1l_1;p_2l_2}(t_1, t_2, t_3, t_4) = \frac{\sqrt{J_{l_2}J_{l_1}}}{\sqrt{(J_L + J_R)(J_L + J_R)}} \times \int dz' \int dz G_f(t_1, z) g_{c;p_2l_2}(z, t_4) G_B(z, z') g_{c;p_1l_1}(t_3, z') G_f(z', t_2)$$

with $g_{c;p_1l_1}(t, t') = \langle t p_1 l_1 | g_c | t' p_1 l_1 \rangle$. For equal times we have

$$\langle c_{p_2\alpha_2\sigma_2l_2}^\dagger(t) c_{p_1\alpha_1\sigma_1l_1}(t) f_{s_2}^\dagger(t) f_{s_1}(t) \rangle_C = i \frac{1}{N} \frac{1}{n_c} \delta_{s_1\sigma_2} \delta_{s_2\sigma_1} \delta_{\alpha_1\alpha_2} F_{p_1l_1;p_2l_2}(t),$$

where the time-ordering for the equal-time limit is defined through $F_{p_1l_1;p_2l_2}(t) = \lim_{t_1, 2, 3, 4 \rightarrow t} F_{p_1l_1;p_2l_2}(t_1, t_2, t_3, t_4)|_{t_1 > t_2 > t_3 > t_4}$. $F_{p_1l_1;p_2l_2}(t)$ can be explicitly evaluated using Langreth rules and making use of the fact that we describe a steady-state. This procedure is straightforward but involved and yields

$$F_{p_1l_1;p_2l_2}(t) = 4i\pi^5 \left[\mathcal{I}_{l_1p_1, l_2p_2}^{(1)} + \mathcal{I}_{l_1p_1, l_2p_2}^{(2)} \right],$$

with

$$\begin{aligned} \mathcal{I}_{p_1, p_2}^{(1)} &= \frac{1}{8} \frac{\sqrt{J_{l_2}J_{l_1}}}{(J_L + J_R)} \int \frac{d\omega}{2\pi} \{ [-i\mathcal{H}[A_{l_2}^{++++}](\omega) + A_{l_2}^{++++}(\omega)] [2\rho_B^+(\omega)] [i\mathcal{H}[A_{l_1}^{++++}](\omega) + A_{l_1}^{++++}(\omega)] \\ &\quad + [-i\mathcal{H}[A_{l_2}^{++--}](\omega) + A_{l_2}^{++--}(\omega)] [-i\rho_B^H(\omega) + \rho_B^-(\omega)] [i\mathcal{H}[A_{l_1}^{++--}](\omega) + A_{l_1}^{++--}(\omega)] \\ &\quad + [-i\mathcal{H}[A_{l_2}^{--++}](\omega) + A_{l_2}^{--++}(\omega)] [i\rho_B^H(\omega) + \rho_B^-(\omega)] [i\mathcal{H}[A_{l_1}^{--++}](\omega) + A_{l_1}^{--++}(\omega)] \} \\ \mathcal{I}_{p_1, p_2}^{(2)} &= \frac{1}{2} \sqrt{J_{l_2}J_{l_1}} \frac{1}{2\pi i} \int \frac{d\omega}{2\pi} \{ [-i\mathcal{H}[A_{l_2}^{++++}](\omega) + A_{l_2}^{++++}(\omega)] A_{l_1}^{--++}(\omega) \\ &\quad - [-i\mathcal{H}[A_{l_2}^{--++}](\omega) + A_{l_2}^{--++}(\omega)] A_{l_1}^{++--}(\omega) \}, \end{aligned}$$

where we defined

$$\begin{aligned} A_l^\Sigma(\omega) &= \int \frac{d\nu}{2\pi} \left[\rho_f^{\Sigma(1)}(\nu) \rho_{c, p_l}^{\Sigma(2)}(\nu - \omega) - \rho_f^{\Sigma(3)}(\nu) \rho_{c, p_l}^{\Sigma(4)}(\nu - \omega) \right], \\ \mathcal{H}[A](\omega) &= -\frac{1}{\pi} P \int d\nu \frac{A(\nu)}{\omega - \nu}. \end{aligned}$$

Currents

The currents of particles and energy through the system are obtained from the change in particle number and energy of *e.g.* the left lead through a continuity equation for the conserved charge (particle number or energy),

$$\begin{aligned} \mathcal{J}_b &= -\partial_t \langle \mathcal{Q}_b(t) \rangle = -i \langle [H(t), \mathcal{Q}_b(t)] \rangle \\ \mathcal{J}_E \rightarrow \mathcal{Q}_E &= H_L = \sum_{p, \alpha\sigma} \varepsilon_{pL} c_{p\alpha\sigma L}^\dagger c_{p\alpha\sigma L} \\ \mathcal{J}_P \rightarrow \mathcal{Q}_P &= N_L = \sum_{p, \alpha\sigma} c_{p\alpha\sigma L}^\dagger c_{p\alpha\sigma L} \end{aligned}$$

Using the identity $[c_\alpha^\dagger c_\beta, c_\gamma^\dagger c_\delta] = \delta_{\beta,\gamma} c_\alpha^\dagger c_\delta - \delta_{\alpha,\delta} c_\gamma^\dagger c_\beta$ and the fact that the Hamiltonian can be decomposed as $H = H_L + H_R + H_J$ with

$$H_J = \frac{1}{N} \sum_{ll'} J_{ll'} \sum_{\sigma\sigma'} \sum_{\alpha} (f_\sigma^\dagger f_{\sigma'} - q \delta_{\sigma\sigma'}) c_{0,\alpha\sigma'l'}^\dagger c_{0\alpha\sigma l},$$

one obtains

$$\begin{aligned} \mathcal{J}_P(t)/M &= 2\sqrt{J_{L,t}J_{R,t}} \operatorname{Re} \left[\frac{1}{n_c^2} \sum_{pp'} F_{Rp',Lp}(t) \right], \\ \mathcal{J}_E(t)/M &= 2\sqrt{J_{L,t}J_{R,t}} \operatorname{Re} \left[\frac{1}{n_c^2} \sum_{pp'} \varepsilon_{pL} F_{Rp',Lp}(t) \right]. \end{aligned}$$

Susceptibility

On the Keldysh contour the impurity spin susceptibility is defined by

$$\chi(z, z') = -i \frac{1}{N} \sum_a \langle T_\gamma S^a(z) S^a(z') \rangle,$$

where T_γ is the time-ordering operator on the Keldysh contour.

For a steady state, we obtain

$$\chi^\pm(\omega) = -\frac{1}{2} \int d\nu \left[\rho_f^+(\nu - \omega) \rho_f^\pm(\nu) - \rho_f^-(\omega - \nu) \rho_f^\mp(\nu) \right],$$

where $\chi_f^\pm(\omega) = -\frac{1}{2\pi i} [\chi^>(\omega) \pm \chi^<(\omega)]$.

Kondo singlet strength

It follows from the Hamiltonian, Eq. (1), that the Kondo term contribution to the total energy is given by

$$\begin{aligned} E_K(t) &= \frac{1}{N} \sum_{ll'} \sum_c J_{ll'} \langle \mathbf{S}(t) \cdot \mathbf{s}_{c,ll'}(t) \rangle \\ &= \kappa \left(\frac{N^2 - 1}{N} \right) \left\{ i \sum_{l_1 l_2} J_{l_1 l_2} \left[\frac{1}{n_c^2} \sum_{p_1 p_2} F_{p_1 l_1; p_2 l_2}(t) \right] \right\} \\ &= -J\kappa \left(\frac{N^2 - 1}{N} \right) \phi_s(t). \end{aligned}$$

This expression can be greatly simplified using the definition of $F_{p_1 l_1; p_2 l_2}(t)$, see previous section. This then yields for the Kondo singlet strength

$$\phi_s = \pi/J w^H(0),$$

with

$$w^-(\omega) = \frac{1}{2} \int d\nu [\sigma_B^+(\nu - \omega) \rho_B^-(\nu) - \sigma_B^-(\nu - \omega) \rho_B^+(\nu)].$$

ADDITIONAL NUMERICAL RESULTS - OTHER VALUES OF r AND κ

In this section we provide further numerical support for our conclusions. Figure S1 shows our results for the parameter set $(r, \kappa) = (0.15, 0.16)$ which is different from the one the results in the paper are based on.

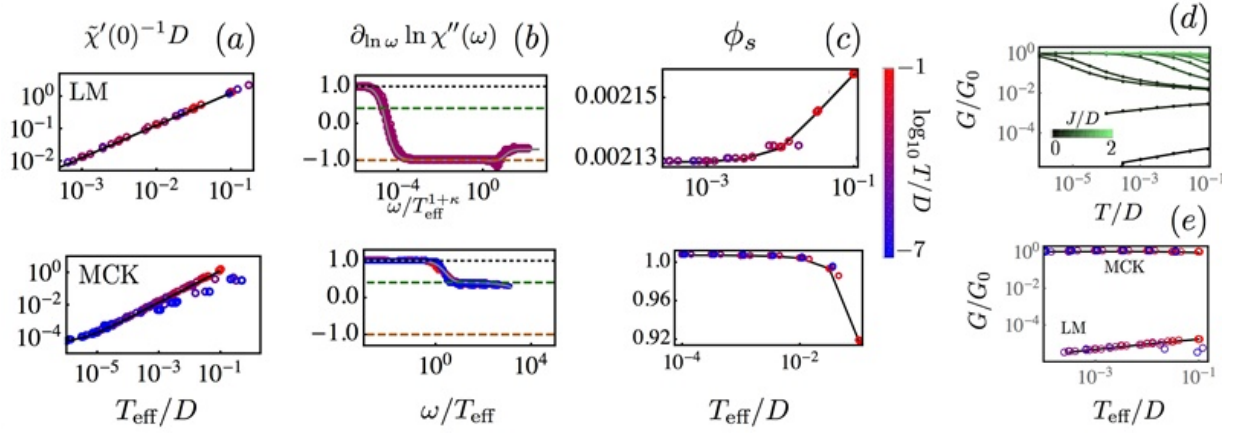


Figure S1. Scaling of different observables with T_{eff} for the different fixed points (the parameters used here differ from those of Figure 3 and 4 of the paper): (a) Inverse static susceptibility $\chi'(0)^{-1}$ vs T_{eff} ; (b) $\partial_{\ln \omega} \ln \chi''(\omega)$ vs ω/T_{eff} ; (c) singlet strength ϕ_s vs T_{eff} . For each fixed point, the equilibrium scaling form (black dashed lines) is compared with the same quantity under non-equilibrium conditions where T is substituted by T_{eff} . (d) Conductance G as a function of temperature computed for the lowest non-zero value of V for several values of J (see color coding). (e) $G = \mathcal{J}_P/V$ vs T_{eff} . for the different fixed points. The equilibrium form is depicted by the black dashed lines. G_0 is defined as the zero-temperature limit of G in the MCK regime.